Linear Algebra & It's Applications by David Lay

book reviewed by Mandy Roberts

David Lay, the author of Linear Algebra & It's Applications, is a distinguished University of Maryland Professor Emeritus. As a top educator of students he holds many honors including founding the National Science Foundation Linear Algebra Curriculum Study Group. In 1994 he received a prestigious award from the Mathematical Association for teaching Mathematics to University students.

I Studying Linear Algebra

The textbook does not ease the reader into thinking about abstraction rather the first couple of sections introduce difficult to grasp concepts such as linear independence, multiple dimensions of space, matrix inverses, and column and null spaces. It is only later in the text that the reader is given sets of exercises to develop his or her understanding of the applications of the concepts. Intermixed with the theory and latter in the text are sections on specific applications of linear algebra like the Leontif Input-Output Model, Markov Chains, and Stochastic Models.

Linear algebra is typically a required course for college students pursuing a degree in Statistics, Mathematics or Engineering. Unfolding the meaning of linear independence is critical for a study in statistics. The memorization of theorems, and the use of proofs is needed for a serious student of Mathematics, while Engineers study linear algebra to develop the principles of mechanics, circuits and other parts of the physical sciences.

II The Fundamentals of Linear Algebra

Linear algebra is about finding solutions to mathematical relationships called equations. The premise of linear algebra is quite simple, however as the number of equations increases the solution sets become more complex. This elementary notion about solution sets developed over time into axioms, theorems and proofs demonstrating the sequences needed to solve increasingly complex systems of equations. The most basic concepts of linear algebra is a familiar concept, one we learned in high school mathematics class. To find the solution of two variables two equations are needed.

$$\begin{cases} x & -2y = & 10 \\ 3x & +y = & 10 \end{cases} \tag{1}$$

To find the solutions of three variables three equations are needed. An example of a system of three equations with three variables is written as follows:

$$\begin{cases} x & +y & +z = & 12 \\ x & +2y & +z = & 15 \\ 5x & +y & +3z = & 34 \end{cases} \tag{2}$$

III Dimensions of Space

Each variable added to an equation is a new dimension in space. An equation with one variable is a point, an equation with two variables lies in 2 dimensional space, an equation with three variables lies in 3 dimensionial space, (etc). As one may surmise the number of dimensions quickly becomes too complicated to comprehend. Try thinking of a shape of a figure in four dimensions.

IV R^n

As one may surmise many dimensions in space cannot be comprehended when one thinks of graphing any equation with more than three variables. The transition from writing down multiple equations to writing a matrix of numbers serves the purpose of simplification. Dropping the equality signs, the addition and subtraction signs means less writing. The system 1 is represented in matrix form by using the coefficients of the variables on the left hand side of two equations and the solution on the right hand side.

$$\begin{bmatrix} 1 & 2 & 10 \\ 3 & 1 & 10 \end{bmatrix} \tag{A}$$

The system 2 is represented in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 1 & 2 & 1 & 15 \\ 5 & 1 & 3 & 34 \end{bmatrix}$$

V Solution Sets

As was said in the first paragraph a system with two equations and two unknowns will yield a solution set. High school algebra taught us to solve for x, substitute the expression in terms of y into x. So a high school student would follow the following steps in the sequence.

$$x = 10 - 2y \qquad \text{solve for x} \tag{1}$$

$$3(10 - 2y) + y = 10 \qquad \text{substitute 1 into the second equation of MatrixA} \tag{2}$$

$$y = 4 \qquad \text{to find the solution set} \tag{3}$$

After studying linear algebra a University student uses a technique called Gaussian elimination to find the solution of the simple Matrix A. Gaussian elimination was discovered in the 19th Century by Johann Carl Friedrich Gauss, a statistician. As Matrices grow in size and complexity the use of a matrix calculator or the use of higher order computer programming is necessary to find the solution set.